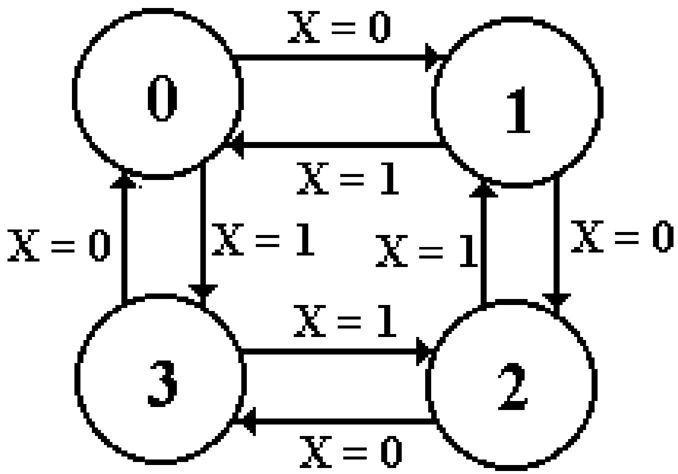
Modulo–4 Up–Down Counter

This is a counter with input.

If X = 0, the device counts up: 0, 1, 2, 3, 0, 1, 2, 3, etc.
If X = 1, the device counts down: 0, 3, 2, 1, 0, 3, 2, 1, etc.

Step 1a: Derive the state diagram and state table for the circuit.



Note two transitions between the state pairs: one is up and one is down.

Step 1b: Derive the State Table

Present State	Next State	
	X = 0	X = 1
0	1	3
1	2	0
2	3	1
3	0	2

This is just a restatement of the state diagram.

Note: Two columns for the "Next State".

Step 2: Count the States and Determine the Flip–Flop Count

Count the States

There are four states for any modulo–4 counter. **N = 4**

The states are simple: 0, 1, 2, and 3.

Calculate the Number of Flip–Flops Required

Let P be the number of flip–flops.

Solve $2^{P-1} < N \leq 2^{P}$. So $2^{P-1} < 4 \leq 2^{P}$ and **P = 2**.

We need two flip–flops.

Step 3: Assign a unique P-bit binary number (state vector) to each state.

Here P = 2, so we are assigning two–bit binary numbers.

Vector is denoted by the binary number Y_1Y_0.

State	2-bit Vector
	Y_1 Y_0
0	0 0
1	0 1
2	1 0
3	1 1

Each state has a unique 2–bit number assigned.

Any other assignment would be absurd.

Step 4: Derive the state transition table and the output table.

There is no computed output, hence no output table.

The state transition table uses the 2–bit state vectors

Present State		Next State	
		X = 0	X = 1
0	00	01	11
1	01	10	00
2	10	11	01
3	11	00	10

Step 5:Separate the state transition table into P tables, one for each flip-flop.

Flip–Flop 1

Flip-Flop 1		
PS	Next State: Y_1	
Y_1Y_0	X = 0	X = 1
0 0	0	1
0 1	1	0
1 0	1	0
1 1	0	1

Flip–Flop 0

Flip-Flop 0: Y_0		
PS	Next State	
Y_1Y_0	X = 0	X = 1
0 0	1	1
0 1	0	0
1 0	1	1
1 1	0	0

Step 6: Decide on the types of flip-flops to use. When in doubt, use all JK's.

Here is the excitation table for a JK flip–flop

Q(T)	Q(T+1)	J	K
0	0	0	d
0	1	1	d
1	0	d	1
1	1	d	0

Step 7: Derive the input table for each flip-flop

Flip–Flop 1

	X = 0			X = 1		
Y_1Y_0	Y_1	J_1	K_1	Y_1	J_1	K_1
0 0	0	0	d	1	1	d
0 1	1	1	d	0	0	d
1 0	1	d	0	0	d	1
1 1	0	d	1	1	d	0

Flip–Flop 0

	X = 0			X = 1		
Y_1Y_0	Y_0	J_0	K_0	Y_1	J_0	K_0
0 0	1	1	d	1	1	d
0 1	0	d	1	0	d	1
1 0	1	1	d	1	1	d
1 1	0	d	1	0	d	1

Question: How do we produce equations for the J's and K's?

Step 8: Derive the input equations for each flip-flop

The equations are based on the present state and the input.

The input X produces a complication.

The simplest match procedure will lead to two equations for each flip–flop input: one for X = 0 and one for X = 1.

Use the “combine rule”

The rule for combining expressions derived separately for X = 0 and X = 1 is

$\overline{X}\bullet$(expression for X= 0) + $X\bullet$(expression for X = 1).

Rationale: Let $F(X) = A\bullet\overline{X} + B\bullet X$

When X = 0, F(X) = A and when X = 1, F(X) = B.

Input Equations for Flip–Flop 1

	X = 0			X = 1		
$Y_1 Y_0$	Y_1	J_1	K_1	Y_1	J_1	K_1
0 0	0	0	d	1	1	d
0 1	1	1	d	0	0	d
1 0	1	d	0	0	d	1
1 1	0	d	1	1	d	0

X = 0: $J_1 = Y_0$, $K_1 = Y_0$

X = 1: $J_1 = Y_0'$, $K_1 = Y_0'$

Apply the "combine rule"

$$J_1 = X' \bullet Y_0 + X \bullet Y_0' = X \oplus Y_0$$

$$K_1 = X' \bullet Y_0 + X \bullet Y_0' = X \oplus Y_0$$

Input Equations for Flip–Flop 0

	X = 0			X = 1		
$Y_1 Y_0$	Y_0	J_0	K_0	Y_1	J_0	K_0
0 0	1	1	d	1	1	d
0 1	0	d	1	0	d	1
1 0	1	1	d	1	1	d
1 1	0	d	1	0	d	1
		$J_0 = 1$ $K_0 = 1$			$J_0 = 1$ $K_0 = 1$	

Apply the "Combine Rule"

$J_0 = X' \bullet 1 + X \bullet 1 = 1$

$K_0 = X' \bullet 1 + X \bullet 1 = 1$

Neither J_0 nor K_0 depend on X. But Y_0 does not depend on X.

Step 9: Summarize the equations by writing them in one place.

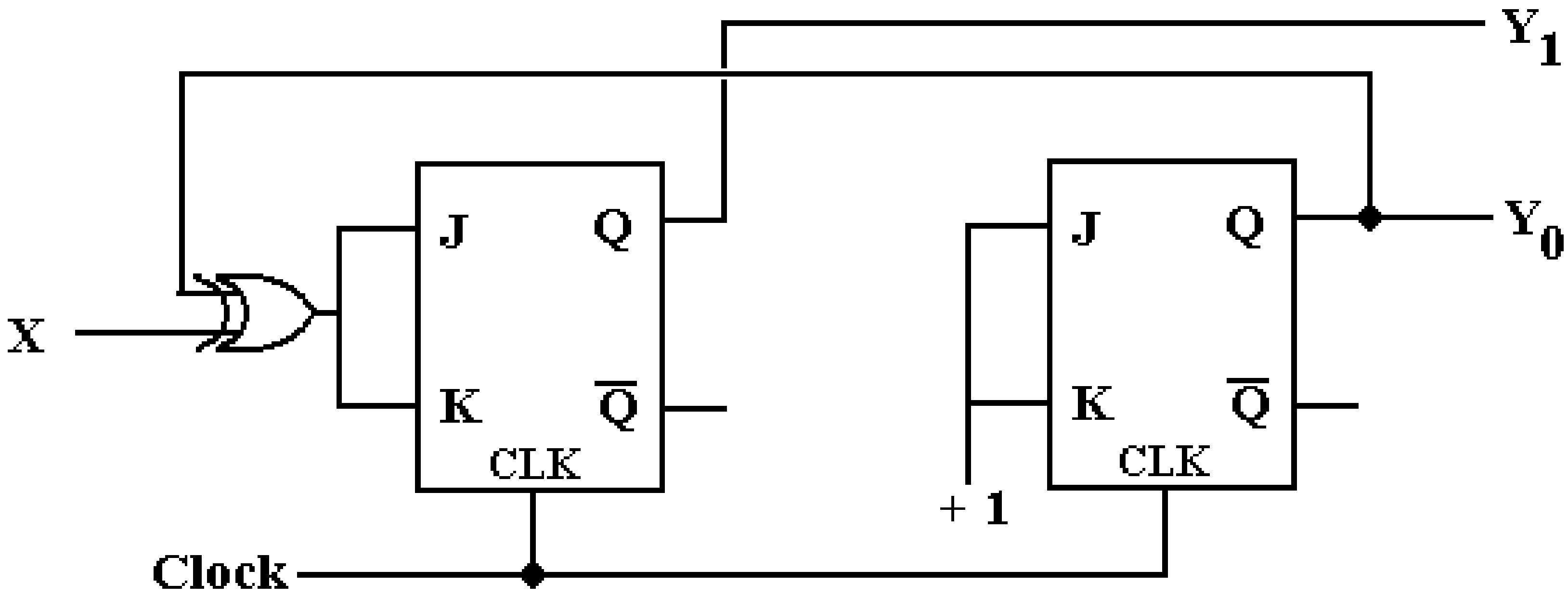
Here they are.

$J_1 = X \oplus Y_0$ $K_1 = X \oplus Y_0$

$J_0 = 1$ $K_0 = 1$

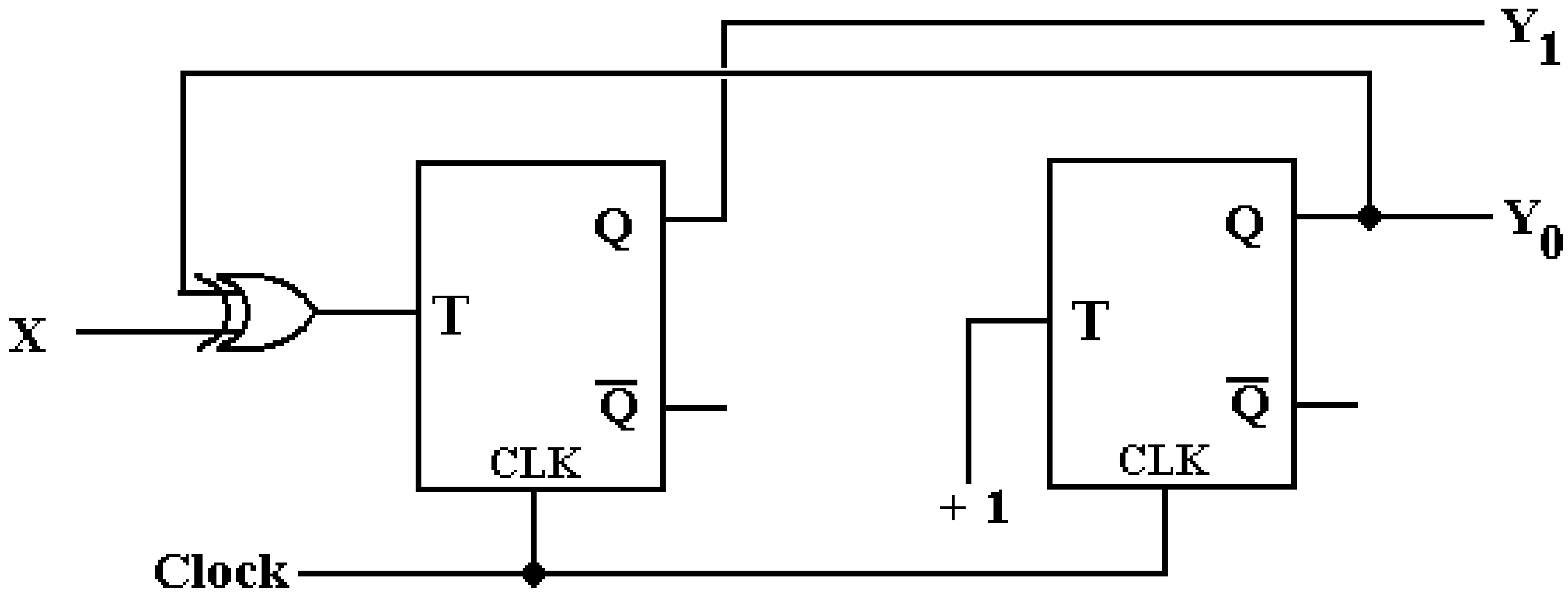
Step 10: Draw the Circuit

As designed, it is:



Step 10: Draw the Circuit

Implemented with T Flip–Flops, it is:



One could also use a 4–register "one hot" design, with the input X used to determine the direction of the shift.